Combined effects of radiation and chemical reaction on MHD flow past a moving plate with Hall current

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Abstract

Influence of radiation and chemical reaction on MHD flow past a moving plate with Hall current is studied here. Earlier, we (2016) have studied unsteady MHD flow in porous media over exponentially accelerated plate with variable wall temperature and mass transfer along with Hall current. To study further, we are changing the model by considering radiation and chemical reaction on flow, and changing geometry of the model. Now, we are taking the plate positioned vertically upward. Laplace method is used to solve the flow model. The results obtained have been analyzed with the help of graphs drawn for different parameters. The numerical values obtained for the drag, Sherwood and Nusselt numbers at boundary have been tabulated. Here too, the results are found to be in agreement with the actual flow.

Keywords: MHD flow, radiation, chemical reaction.

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1. Introduction

MHD flow problems over an impulsively started vertical plate play important role in many branches of science and technology. The effects of radiation and chemical reaction on MHD flow are also significant in many cases. Some such problems already studied are mentioned here. The Hall effect in the viscous flow of ionized gas between parallel plates under transverse magnetic field was studied by Sato (1961). Mazumder and Deka (2007) have considered MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Ibrahim and Makinde (2010) have investigated chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Reddy et al (2014) have studied MHD flow considering free convection over a porous plate. Balla and Naikoti (2015) have considered unsteady MHD flow with convective heat and mass diffusion. MHD flow over a stretching surface was analyzed by Jonnadula et al (2015). Malapati and Polarapu (2015) have worked on MHD flow with natural convection. Unsteady MHD flow in porous media was investigated by us (2016). Rajput and Kanaujia (2016) have worked on chemical reaction in MHD flow past a vertical plate with mass diffusion and constant wall temperature with Hall current. The motive of this study is to analyze the combined effects of radiation and chemical reaction on fluid flow over a moving plate in the presence of transversely applied uniform magnetic field and Hall current. The fluid model under consideration has been solved by Laplace transform method. The model contains equations of motion, diffusion equation and equation of energy. To analyze the solution of the model, desirable sets of the values of the parameters have been considered. The numerical data obtained is discussed with the help of graphs and tables.

2. Mathematical Analysis

The physical model is shown in Figure-1
Consider an unsteady MHD flow past an impulsively started vertical plate. The fluid is electrically conducting. The x axis is along the vertical plate and z is perpendicular to it. Thus the z axis lies in the horizontal plane. The uniform magnetic field \( B_0 \) is applied perpendicular to the fluid. Initially it has been considered that the plate as well as the fluid is at the same temperature \( T_\infty \). The species concentration in the fluid is taken as \( C_\infty \). At time \( t > 0 \), the plate starts moving with a velocity \( u_0 \). The wall temperature \( T_w \) and the concentration \( C_w \) in the boundary region are raised in proportion with time. So, under above assumptions, the governing equations are as follows:

Momentum Equation
\[
\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + g\beta (T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho (1 + m^2)}
\]
(1)

Concentration Equation
\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} - K_c (C - C_\infty)
\]
(3)

Energy Equation
\[
\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z}
\]
(4)

Here \( K_c \) is chemical parameter, \( u \) and \( v \) are the primary and secondary velocities along x and z directions respectively. Other symbols used have the following description:

- \( C \) - species concentration in the fluid,
- \( m \) - the Hall current parameter,
- \( T \) - temperature of the fluid,
- \( T_w \) - temperature of the plate at \( z=0 \),
- \( C_w \) - species concentration at the plate \( z=0 \),
- \( B_0 \) - the uniform magnetic field,
- \( \sigma \) - electrical conductivity.
- \( \beta^* \) - volumetric coefficient of concentration expansion,
- \( \nu \) - the kinematic viscosity,
- \( \rho \) - the density,
- \( C_p \) - the specific heat at constant pressure,
- \( g \) - the acceleration due to gravity,
- \( \beta \) - volumetric coefficient of thermal expansion,
- \( t \) - time,
- \( k \) - thermal conductivity of the fluid,
- \( D \) - the mass diffusion coefficient.

The boundary conditions taken are as under:

\[
t \leq 0 : u = v = 0, \ T = T_\infty, \ C = C_\infty, \ \text{for every } z.
\]
\[
t > 0 : u = u_0, \ v = 0, \ T = T + (T_w - T_\infty)A, \ C = C_w + (C_w - C_\infty)A, \ \text{at } z=0.
\]
(5)
By using Rosseland approximation (Brewster (1992)), the radiative heat flux $q_r$ is given by

$$\frac{\partial q_r}{\partial z} = -4a^*\sigma(T_w^4 - T^4)$$

where $a^*$ is absorption constant.

The boundary conditions (5) become:

$$\bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w \text{ as } \bar{z} \to \infty.$$
\[ \frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu-v)}{1+m^2} \]  
(16)

\[ \frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} - K_0 C \]  
(17)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - \frac{R \theta}{P_r} \]  
(18)

\[ t \leq 0 : u = v = \theta = C = 0, \quad \text{for every } z. \]
\[ t > 0 : u = v = 0, \quad \theta = t, \quad C = t, \quad \text{at } z=0. \]
\[ u \to 0, \quad v \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } z \to \infty. \]

Writing the equations (15) and (16) in combined form (using \( q = u + i v \))

\[ \frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r \theta + G_m C - qa \]  
(20)

\[ \frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} - K_0 C \]  
(21)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - \frac{R \theta}{P_r} \]  
(22)

The boundary conditions (19) are reduced to:

\[ t \leq 0 : q = \theta = C = 0, \quad \text{for every } z. \]
\[ t > 0 : q = 1, \theta = t, C = t, \quad \text{at } z=0. \]
\[ q \to 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } z \to \infty. \]

The equations (20) to (22), using equation (23), are solved by the Laplace method. The solution obtained is as under:

\[ q = \frac{1}{2} \exp(-\sqrt{a}z) A_{33} + \frac{G_r}{4(a-R)^2} (\exp(-\sqrt{a}z)(2Ra_1 - 2atA_1 + z\sqrt{a}A_2 + 2A_t(P_r + 1)) - \frac{A_2z}{\sqrt{a}}(at - Rt + P_r - 1)) + \frac{G_m}{4(\alpha - K_0S_c)} (\exp(-\sqrt{a}z)(z\sqrt{a}A_2 - 2atA_1 - 2A_r(S_c - 1) + 2tA_tK_0S_c) - \frac{z \exp(-\sqrt{a}z)A_rK_0S_c}{\sqrt{a}} - 2A_tA_1S_c - 2A_tA_r(S_c - 1) + \exp(-z\sqrt{S_cK_0}))( - \frac{aA_2zS_c}{\sqrt{K_0}}) \right) \]  
(24)

\[ \theta = e^{\frac{\sqrt{a}z}{4\sqrt{a}}} \left[ \text{erfc}\left(\frac{-\sqrt{a}Rt + zP}{\sqrt{P_r t}}\right)(2\sqrt{R}t - zR) + e^{\frac{z\sqrt{a}c}{2\sqrt{R}}} \text{erfc}\left(\frac{2\sqrt{R}t + zP}{\sqrt{P_r t}}\right)(2\sqrt{R}t + zR)\right], \]

\[ C = \frac{e^{-z\sqrt{S_cK_0}}}{4\sqrt{K_0}} \left[ \text{erfc}\left(\frac{z\sqrt{S_c} - 2R}{2\sqrt{t}}\right)(-z\sqrt{S_c} + 2R) + e^{2z\sqrt{S_cK_0}} \text{erfc}\left(\frac{z\sqrt{S_c} + 2R}{2\sqrt{t}}\right)(z\sqrt{S_c} + 2R) \right]. \]

The expressions for the symbols involved in the above solution are given in the appendix.

3. Skin Friction
The dimensionless skin-friction at the surface is
\[ \left( \frac{dq}{dc} \right)_{z=0} = \tau_x + i \tau_y. \]

The numerical values of \( \tau_x \) and \( \tau_y \), for different parameters, are given in table-1.

4. Nusselt number

The dimensionless Nusselt number is given by
\[ Nu = \left( \frac{\partial \theta}{\partial z} \right)_{z=0} = \text{erfc}[\sqrt{\frac{R}{\sqrt{tP_r}}}](\sqrt{\frac{R}{2}} t + \frac{P_r}{4\sqrt{R}}) - \text{erfc}[-\sqrt{\frac{R}{\sqrt{tP_r}}}](-\sqrt{\frac{R}{2}} t + \frac{P_r}{4\sqrt{R}}) - \frac{R t}{\sqrt{\pi}}. \]

5. Sherwood number

The dimensionless Sherwood number at the surface is
\[ S_h = \left( \frac{\partial C}{\partial z} \right)_{z=0} = \text{erfc}[\sqrt{\frac{1}{K_0}}](\sqrt{\frac{1}{4K_0}} S_c - t \sqrt{\frac{S_c K_0}{2}}) + \sqrt{S_c} \text{erfc}[\sqrt{\frac{1}{K_0}}](\frac{1}{4\sqrt{K_0}} S_c t \sqrt{\frac{S_c K_0}{2}}) - \frac{t S_c K_0}{\sqrt{\pi K_0}}. \]

6. Results and discussion

The study is carried out to examine the effects of radiation with chemical reaction on unsteady MHD flow past a moving plate with variable wall temperature and mass diffusion in the presence of Hall current. The behavior of other parameters like magnetic parameter, Hall current and thermal buoyancy is almost similar to the earlier model studied by us (2016). The analytical results are shown in figures 2 to 7. The numerical values of skin-friction, Sherwood number and Nusselt number are presented in Table-1, 2 and 3, respectively. Chemical reaction effect on fluid flow behavior is shown by figures 2 and 3. It is seen here, when chemical reaction parameter \( K_0 \) increases, \( u \) and \( v \) decrease throughout near the surface. Figures 4 and 5 indicate that effect of radiation in the flow near the plate tends to accelerate velocities. This is due to the fact that the large values of radiation parameter tend to accelerate velocity of the fluid in the region near the surface of the plate. Further, it is noticed that the temperature and concentration of the fluid near the plate decrease when radiation and chemical reaction parameters are increased (figures 6 and 7).

Skin friction is given in table 1. The values of Skin friction \( \tau_x \) and \( \tau_y \) increase with the increase in \( R \) and decrease with \( K_0 \). Sherwood number is given in table 2. The value of \( S_h \) decreases with the increase in \( K_0, S_c \) and \( t \). Nusselt number is given in table 3. The value of \( Nu \) decreases with increase in \( P_r, R \) and \( t \).
Figure 3: $v$ vs $z$

$M = 2, m = 1, P_r = 0.71, S_c = 2.01, G_m = 100, K_0 = 1, t = 0.4$

Figure 4: $u$ vs $z$

$M = 2, m = 1, P_r = 0.71, S_c = 2.01, G_m = 100, G_r = 10, K_0 = 1, t = 0.4$

Figure 5: $v$ vs $z$

$M = 2, m = 1, P_r = 0.71, S_c = 2.01, G_m = 100, G_r = 10, K_0 = 1, t = 0.4$
Table 1: Skin-friction for different parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$m$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$Gm$</th>
<th>$Gr$</th>
<th>$R$</th>
<th>$K_0$</th>
<th>$t$</th>
<th>$\theta$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>04</td>
<td>01</td>
<td>0.4</td>
<td>01215.7783</td>
<td>01284.2268</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>08</td>
<td>01</td>
<td>0.4</td>
<td>01214.7526</td>
<td>01284.2078</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>04</td>
<td>05</td>
<td>0.4</td>
<td>01214.7722</td>
<td>01284.2217</td>
</tr>
</tbody>
</table>

Table 2: Sherwood number for different parameters.

<table>
<thead>
<tr>
<th>$K_0$</th>
<th>$Sc$</th>
<th>$T$</th>
<th>$S_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>2.01</td>
<td>0.2</td>
<td>-0.762200</td>
</tr>
<tr>
<td>05</td>
<td>2.01</td>
<td>0.2</td>
<td>-0.933049</td>
</tr>
<tr>
<td>10</td>
<td>2.01</td>
<td>0.2</td>
<td>-1.118240</td>
</tr>
</tbody>
</table>
7. Conclusion

The results obtained are in agreement with the usual flow. It is observed that the velocities near the plate surface are increased with radiation parameter, and decrease with chemical reaction. However the drag decreases with chemical reaction. Also the values of \( S_h \) and \( N_u \) decrease with \( K_0 \) and \( R \). It is as per expected flow behavior.

Appendix:

\[
a = \frac{M}{1 + m^2}, \quad A = \frac{u_0^2}{v}, \quad A_1 = (1 + A_{12} + e^{2\sqrt{ac}} (1 - A_{13})), \quad A_2 = (1 + A_{12} - e^{2\sqrt{ac}} (1 - A_{13})),
\]

\[
A_0 = (A_{14} - 1 + A_{29} (A_{15} - 1)), \quad A_4 = (A_{16} - 1 + A_{29} (A_{17} - 1)), \quad A_5 = (A_{18} - 1 + A_{29} (A_{19} - 1)),
\]

\[
A_6 = (A_{20} - 1 + A_{29} (A_{21} - 1)), \quad A_7 = (e^{2\sqrt{a}S_c} (A_{23} - 1) - A_{22} - 1)), \quad A_8 = (e^{2\sqrt{a}S_c} (A_{23} - 1) + A_{22} + 1)),
\]

\[
A_9 = (A_{30} (A_{25} - 1) - A_{24} - 1), \quad A_{10} = (1 - A_{18} + A_{29} (A_{19} - 1)), \quad A_{11} = AbS[z]Abs[P_r], \quad A_{12} = erf\left[\frac{1}{2}\sqrt{at} - z\right],
\]

\[
A_{13} = erf\left[\frac{1}{2}\sqrt{at} + z\right], \quad A_{14} = erf\left[\frac{1}{2}\sqrt{at} \left(\frac{aP_r - R}{P_r - 1}\right)\right], \quad A_{15} = erf\left[\frac{1}{2}\sqrt{at} \left(\frac{aP_r - R}{P_r - 1}\right)\right],
\]

\[
A_{16} = erf\left[\frac{1}{2}\sqrt{at} \left(\frac{(a - K_0)S_c}{S_c - 1}\right)\right], \quad A_{17} = erf\left[\frac{1}{2}\sqrt{at} \left(\frac{(a - K_0)S_c}{S_c - 1}\right)\right], \quad A_{18} = erf\left[\frac{A_{12}}{2\sqrt{t}P_r}\right],
\]

\[
A_{19} = erf\left[\frac{A_{12}}{2\sqrt{t}P_r}\right], \quad A_{20} = erf\left[\frac{A_{12}}{2\sqrt{t}P_r}\right], \quad A_{21} = erf\left[\frac{A_{12}}{2\sqrt{t}P_r}\right],
\]

\[
A_{22} = erf\left[\frac{1}{2}\sqrt{2t} \left(\frac{(a - K_0)S_c}{S_c - 1} - z\sqrt{S_c}\right)\right], \quad A_{23} = erf\left[\frac{1}{2}\sqrt{2t} \left(\frac{(a - K_0)S_c}{S_c - 1} + z\sqrt{S_c}\right)\right], \quad A_{24} = erf\left[\frac{1}{2}\sqrt{2t} \left(\frac{(a - K_0)S_c}{S_c - 1} - z\sqrt{S_c}\right)\right],
\]

\[
A_{25} = erf\left[\frac{1}{2}\sqrt{2t} \left(\frac{(a - K_0)S_c}{S_c - 1} + z\sqrt{S_c}\right)\right], \quad A_{26} = \exp\left(\frac{at}{P_r - 1} - \frac{Rt}{P_r - 1} - z\sqrt{aP_r - R}\right),
\]

\[
A_{27} = \exp\left(\frac{at}{S_c - 1} - \frac{tS_c K_0}{S_c - 1} - z\sqrt{aP_r - R}\right), \quad A_{28} = \frac{1}{A_{31}} \exp\left(\frac{at}{P_r - 1} - \frac{Rt}{P_r - 1}\right), \quad A_{29} = \exp\left(2z\sqrt{\frac{-R + aP_r}{P_r - 1}}\right),
\]

\[
A_{30} = \exp(2z\sqrt{\frac{(a - K_0)S_c}{S_c - 1}}),
\]

\[
R_{t}, \quad \nu = \frac{2}{3}Pr, \quad \nu = \frac{2}{3}Pr, \quad \nu = \frac{2}{3}Pr
\]
\[ A_{31} = \exp(2A\text{bs}[z]\sqrt{\frac{P_r(aP_r - R)}{P_r - 1}}), \quad A_{32} = \exp(2A\text{bs}[z]\sqrt{P_r R}), \quad A_{33} = 1 + A_{34} + \exp(2\sqrt{a}z)A_{35}, \]
\[ A_{34} = \text{erf}\left[\frac{1}{2\sqrt{t}}(2\sqrt{at} - z)\right], \quad A_{35} = \text{erfc}\left[\frac{1}{2\sqrt{t}}(2\sqrt{at} + z)\right], \]

References


Biographical notes

Dr Uday Singh Rajput is a faculty member in the department of mathematics and astronomy, Lucknow University, India. He has more than 25 years of teaching experience at UG and PG levels and also guided students for PhD degree. He has published more than 70 research articles. His research areas include MHD flows, Graph Theory and Operations Research.

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